AC21007: Haskell Lecture 6
Tail Recursion, Algebraic Data Types, Type Classes

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Recapitulation

- Sorting algorithms
  - Selection Sort
  - Insertion Sort
  - Bubble Sort

Tail recursion

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sum []      = 0
sum (x:xs)  = x + (sum xs)
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```

- A tail-recursive version - we use an additional accumulator `acc`:

```
sumAux :: [Int] -> Int -> Int
sumAux []     acc = acc {- + 0 -}
sumAux (x:xs) acc = sumAux (acc + x) xs

sum :: [Int] -> Int
sum xs = sumAux xs 0
```

Tail recursion - Fibonacci numbers

- Fibonacci numbers: $F_n = \begin{cases} = 0 & n = 0 \\ = 1 & n = 1 \\ = F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$

  $0, 1, 1, 2, 3, 5, 8, 13, \ldots$

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- Haskell implementation is straightforward:

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fib :: Integer -> Integer
fib 0 = 0
fib 1 = 1
fib n = fib (n - 1) + fib (n - 2)
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- Can we turn this into a tail-recursive function?

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- An implementation:

```haskell
fibHelper :: Int -> Int -> Int -> Int
fibHelper 0 val1   val2    = val1
fibHelper 1 val1   val2    = val2
fibHelper n val1   val2    =
      fibHelper (n - 1) val2 (val1 + val2)

fib :: Int -> Int
fib n = fibHelper n 0 1
```

Tail recursion and folds

- We already saw folds - "schemes" of recursive functions
- We know that e.g. `sum` can be expressed as a fold:

```haskell
sum :: [Int] -> Int
sum xs = foldl (+) 0 xs
```

or

```haskell
sum' :: [Int] -> Int
sum' = foldr (+) 0
```

- Is either of these tail-recursive?

Tail recursion and folds (cont.)

- recall recursive steps of `foldr` and `foldl`:

```
foldr f z (x:xs) = f x (foldr f z)

...

foldl f z (x:xs) = let z' = z `f` x
                   in foldl f z' xs
```

Tail recursion and folds (cont.)

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- `foldr` is *not* tail-recursive but `foldl` is tail recursive!

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- We already know `Bool`:

```
data Bool = False | True
```

  `Bool` is a type with two constructors: `True` and `False`.
- Similarly we can define e.g.:

```
data Suits = Spades
           | Hearts
           | Diamonds
           | Clubs
```

Algebraic Data Types (cont.)

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- Constructors may contain *fields* of certain type, e.g.:

```
data MyPair = MyPair Int Char
```

Note that the name of a type and a name of it's constructor can be the same. A value of type `MyPair`:

```
myPairVal :: MyPair
myPairVal = MyPair 1 'c'
```

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- Values of algebraic data types are constructed in the same way as values of lists and tuples.

Algebraic Data Types (cont.)

- We can also pattern-match on data type values in function definitions and let-bindings in the very same way as with lists and tuples:

```
incMyPair :: MyPair -> MyPair
incMyPair (MyPair i c) = MyPair (i + 1) c
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```

- . . . but tuple type is more general – `(a, b)` for any types `a` and `b`

Algebraic Data Types (cont.)

- Data types may be polymorphic in fields of constructors,

```
data Pair a b = Pair a b
```

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- And we can combine all of the above:

```haskell
data CrazyType a b c
    = NoParamCtor
    | MonoMorphicCtor1 Int
    | MonoCtor2 String [Char] Bool
    | MonoCtor3 MyPair
    | PolyCtor1 a b
    | PolyCtor2 a Int
    | PolyCtor3 (Pair c Int)
    | PolyCtor4 (c -> a, Int)
```

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Some old ADT's ...

- The list type is just an ordinary type, the only special thing is syntactic sugar for "[]" and "(:)":

```
data List a = Nil | Cons a (List a)

length' :: List a -> Int
length' Nil           = 0
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- And the same for tuples, as we already saw:

```haskell
data Pair a b = Pair a b

fst' :: Pair a b -> a
fst' (Pair x _) = x

snd' :: Pair a b -> b
snd' (Pair _ y) = y
```

. . . and some new

- Sometimes, we need an extra value:

```
data Maybe a = Nothing | Just a

safeHead :: [a] -> Maybe a
safeHead []     = Nothing
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- ADT representing binary trees (values are only in leafs):

```haskell
data BinTree a
    = Leaf a
    | Node (BinTree a) (BinTree a)

myTree :: BinTree Char
myTree = Node (Leaf 'a') (Leaf 'b')
```

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- `Ord a` is a type class *constraint*.

Type Classes (cont.)

- We can define a *class* of types and specify which functions (called class *methods*) are available for types of this class (i.e. type class behaves as an interface), e.g.:

```
class Ord a where
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- We can specify, that a type is and *instance* of a class – we provide an implementation of class functions for this type:

```
instance Ord Int where
    x <= y = primitiveIntComparison x y

instance Ord Float where
    x <= y = primitiveFloatComparison x y
```

Type Classes (cont.)

- Instance definitions can itself be constrained and do recursively compose. Recall our ADT `List`:

```
instance Ord a => Ord (List a) where
    Nil <= _        = True
    (List x xs) <= (List y ys) =
        if (x <= y)
            then if (x == y)
                then xs <= ys
                else True
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- And there is a similar instance for `[a]`
- That means that if we provide instance `Ord OurData` we get `Ord [OurData]` for free.

Type Classes (cont.)

- Some standard Haskell type classes:
  - `Eq a` – types with equality, `(==)`
  - `Ord a` – ordered types, `(<)`, `(<=)`
  - `Show a` – types that can be pretty printed using `show`
  - `Num a` – numeric types - `(+)`, `(-)`, `(*)`, `abs`, `signum`
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  - `Num a` – numeric types - `(+)`, `(-)`, `(*)`, `abs`, `signum`
  - And many more ...
- Now we can fully understand type of e.g. `(+)`:

```
GHCi, version 7.10.3:
Prelude> :t (+)

(+) :: Num a => a -> a -> a
```

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  ```

- **Best Practice:** Do provide top-level types – types document functions, and help compiler produce simpler error messages

Next time

- Monday the the 22th of February, 2-3PM, Dalhousie 3G05 LT2
- More sorting algorithms
  - Quick Sort
  - Merge Sort
- IO in Haskell (Monads)